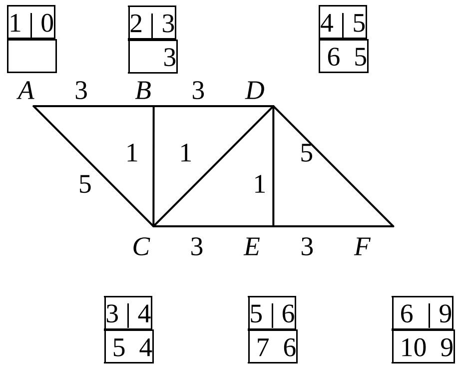
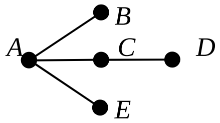
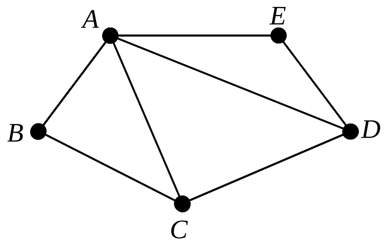
4736 Decision Mathematics 1

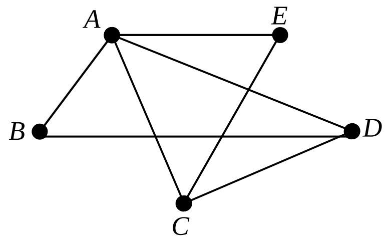
				TO BE ANSWERED ON INSERT	
1	**(i)**	 Path: *A – B – C – D – E – F* Weight: 9	M1 A1 B1 B1 B1	Evidence of updating at *C*, *D*, *E* or *F* All temporary labels correct, with no extras All permanent labels correct cao cao	**[5]**
	(ii)	Total weight of all arcs = 25 Only odd nodes are *B* and *E*. Least weight path joining *B* to *E* is *B – C – E* = 3. Weight: 28 Route: (example) *A–B–D–F–E–C–B–C–D–E – D – C - A*	B1 M1 A1 B1	Total weight = 25 (may be implied from weight) *B* to *E* = 3 28 (cao) A valid closed route that uses *BC*, *CD* and *DE* twice and all other arcs once	**[4]**
	(iii)	*A – B – E – F* Graph is now Eulerian, so no need to repeat arcs	B1 B1	cao Eulerian (or equivalent)	**[2]**
				Total =	**11**

2	**(i)**	A graph cannot have an odd number of odd vertices (nodes)	B1	Or equivalent (eg $3\times5 = 15 \Rightarrow 7\frac{1}{2}$ arcs) Not from a diagram of a specific case	**[1]**
	(ii)	It has exactly two odd nodes eg *C A B C D E A D*	B1 B1	2 odd nodes A valid semi-Eulerian trail	**[2]**
	(iii)	*AE* = 2 *AC* = 3 *AB* = 5 *CD* = 7  Weight = 17	B1 B1 B1	Correct tree (vertices must be labelled) Order of choosing arcs in a valid application of Prim, starting at *A* (working shown on a network or matrix) 17	**[3]**
	(iv)	Lower bound = 29 *A – E – D – F – C – B – A* = 34 *F – C – A – E – D* and *F –D – C – A - E* Vertex *B* is missed out	B1 M1 A1 B1	29 or 12 + their tree weight from **(iii)** *A – E – D – F – C –* 34, from correct working seen Correctly explaining why method fails, need to have explicitly considered both cases	**[4]**
				Total =	**10**

For reference

(ii)

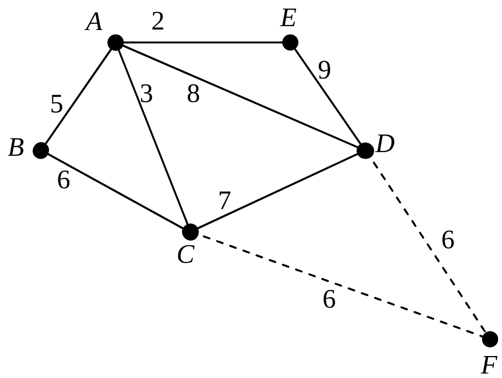


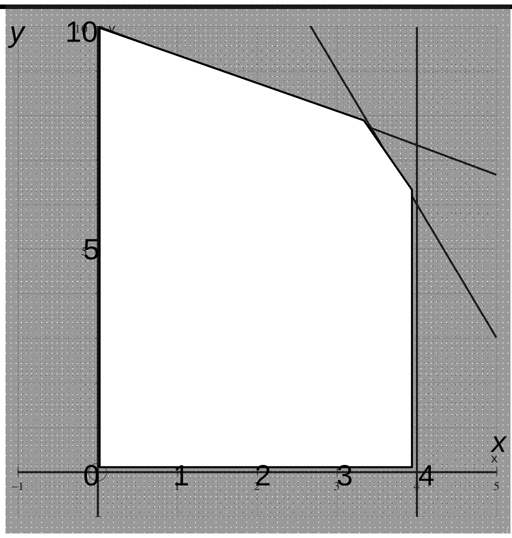


(iii) (iv)

	A	B	C	D	E
A	-	5	3	8	2
B	5	-	6	-	-
C	3	6	-	7	-
D	8	-	7	-	9
E	2	-	-	9	-

CF = 6
DF = 6



3 (i)	x = number of clients who use program X y = number of clients who use program Y	B1	Number of clients on X and Y, respectively	**[1]**
(ii)	Spin cycle: $30x + 10y \leq 180$ $\Rightarrow 3x + y \leq 18$ Rower: $10x \leq 40$ $\Rightarrow x \leq 4$ Free weights: $20x + 30y \leq 300$ $\Rightarrow 2x + 3y \leq 30$	B1 B1 B1	$3x + y \leq 18$, or equivalent, simplified $x \leq 4$, or equivalent, simplified $2x + 3y \leq 30$, or equivalent, simplified Allow use of slack variables instead of inequalities	**[3]**
(iii)	Both must take non-negative integer values	B1	Non-negative and integer Accept $x + y \leq 12$ as an alternative answer	**[1]**
(iv)		B1 M1 A1	Axes scaled and labelled appropriately (on graph paper) Boundaries of their three constraints shown correctly (non-negativity may be missed) Correct graph with correct shading or feasible region correct and clearly identified (non-negativity may be missed) (cao)	**[3]**
	Checking vertices or using a profit line $(4, 6) \rightarrow 72$ $(3\frac{3}{7}, 7\frac{5}{7}) \rightarrow 77\frac{1}{7}$ or $(24/7, 54/7) \rightarrow 77\frac{1}{7}$ $(0, 10) \rightarrow 60$ $(4, 0) \rightarrow 36$ Checking other feasible integer points near (non-integer) optimum for continuous problem $(3, 8) \rightarrow 75$ Put 3 clients on program X, 8 on program Y and 1 on program Z	 M1 M1 A1	Follow through their graph if possible $x = 3.4$, $y = 7.7$ may be implied from (3, 8) Could be implied from identifying point (3, 8) in any form cao, in context and including program Z	**[3]**
			Total =	**11**

4 (i)

A	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*
A	*A*	*A*	*A*	*A*	*D*	*D*	*D*	*D*	*C*
C	*C*	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*B*

B1 — 15 *A*'s, 4 *D*'s, 3 *C*'s, 8*B*'s (but not just *A D C B*)

Box 1	*A A A A A*
Box 2	*A A A A A*
Box 3	*A A A A A*
Box 4	*D D D D C C C B*
	B B B B B B B

M1 — Three boxes each containing *A A A A A* (or shown using weights)

M1 — A box containing all the rest

A1 — Completely correct, including order of packing into boxes

Cannot fit all the items into box 4
There is only room for one *B* in a box

B1 — Any identification of a (specific) volume conflict **[5]**

(ii)

B	*B*	*B*	*B*	*B*	*B*	*B*	*B*	*C*	*C*
C	*D*	*D*	*D*	*D*	*A*	*A*	*A*	*A*	*A*
A	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*	*A*

B1 — 8 *B*'s, 3 *C*'s, 4 *D*'s, 15 *A*'s (but not just *B C D A*)

Box 1	*B D A A*
Box 2	*B D A A*
Box 3	*B D A A*
Box 4	*B D A A*
Box 5	*B A A A A A A*
Box 6	*B A*
Box 7	*B*
Box 8	*B*
Box 9	*C C C*

M1 — Four boxes each containing *B D A A* (in any order)

M1 — Using exactly 9 boxes, the first eight of which each contain a *B* (with or without other items) and the ninth contains three *C*'s.

A1 — Completely correct, including order of packing into boxes

Box 5 is over the weight limit
More than five A's is too heavy for one box

B1 — Any identification of a (specific) weight conflict **[5]**

(iii)

Items may be the wrong shape for the boxes eg too tall

B1 — Reference to shape, height, etc. but not practical issues connected with the food **[1]**

Total = 11

For reference

Item type	*A*	*B*	*C*	*D*
Number to be packed	15	8	3	4
Length (cm)	10	40	20	10
Width (cm)	10	30	50	40
Height (cm)	10	20	10	10
Volume (cm^3)	1 000	24 000	10 000	4 000
Weight (g)	1 000	250	300	400

<table>
<tr><td>5</td><td>(i)</td><td>Minimise $2a - 3b + c + 18$
$\Rightarrow$ minimise $2(20-x) - 3(10-y) + (8-z) + 18$
$\Rightarrow$ minimise $-2x + 3y - z$
$\Rightarrow$ maximise $2x - 3y + z$ (given)

$a + b - c \geq 14$
$\Rightarrow (20-x) + (10-y) - (8-z) \geq 14$
$\Rightarrow x + y - z \leq 8$ (given)

$-2a + 3c \leq 50$
$\Rightarrow -2(20-x) + 3(8-z) \leq 50$
$\Rightarrow 2x - 3z \leq 66$ (given)

$10 + 4a \geq 5b$
$\Rightarrow 10 + 4(20-x) \geq 5(10-y)$
$\Rightarrow 4x - 5y \leq 40$ (given)

$a \leq 20 \Rightarrow 20-x \leq 20 \Rightarrow x \geq 0$
$b \leq 10 \Rightarrow 10-y \leq 10 \Rightarrow y \geq 0$
$c \leq 8 \Rightarrow 8-z \leq 8 \Rightarrow z \geq 0$</td><td>B1

M1

A1</td><td>(Constant has no effect on slope of objective)
Replacing a, b and c in objective to get $-2x + 3y - z$
(Condone omission of conversion to maximisation here)

Replacing a, b and c in the first three constraints to get given expressions

Showing how $a \leq 20$, $b \leq 10$, $c \leq 8$ give $x \geq 0$, $y \geq 0$, $z \geq 0$</td><td>[3]</td></tr>
<tr><td></td><td>(ii)</td><td>

P	x	y	z	s	t	u	RHS
1	-2	3	-1	0	0	0	0
0	1	1	-1	1	0	0	8
0	2	0	-3	0	1	0	66
0	4	-5	0	0	0	1	40

</td><td>M1
A1</td><td>Constraint rows correct, with three slack variable columns
Objective row correct</td><td>[2]</td></tr>
<tr><td></td><td></td><td>x and z columns have negative entries in objective row, but z column has no positive entries in constraint rows, so pivot on x col
$8\div1 = 8$; $66\div2 = 33$; $40\div4 = 10$
Least ratio is $8\div1$, so pivot on 1 from x col</td><td>M1

A1</td><td>Choosing to pivot on x column
(may be implied from pivot choice)
Calculations seen or referred to and correct pivot choice made (cao)</td><td>[2]</td></tr>
<tr><td></td><td></td><td>New row 2 = row 2
New row 1 = row 1 + 2(new row 2)
New row 3 = row 3 – 2(new row 2)
New row 4 = row 4 – 4(new row 2)</td><td>M1

A1</td><td>Pivot row unchanged (may be implied) or follow through for their +ve pivot
Calculations for other rows shown (cao)</td><td>[2]</td></tr>
<tr><td></td><td></td><td>

P	x	y	z	s	t	u	RHS
1	0	5	-3	2	0	0	16
0	1	1	-1	1	0	0	8
0	0	-2	-1	-2	1	0	50
0	0	-9	4	-4	0	1	8

</td><td>M1

A1</td><td>An augmented tableau with three basis columns, non-negative values in final column and value of objective having not decreased
Correct tableau after one iteration (cao)</td><td>[2]</td></tr>
<tr><td></td><td></td><td>$x = 8$, $y = 0$, $z = 0$

$x = 8 \Rightarrow a = 20 - 8 = 12$
$y = 0 \Rightarrow b = 10 - 0 = 10$
$z = 0 \Rightarrow c = 8 - 0 = 8$</td><td>B1

M1

A1</td><td>Non-negative values for x, y and z from their tableau
Putting their values for x, y and z into $a = 20 - x$, $b = 10 - y$ and $c = 8 - z$
Correct values for a, b and c, from their non-negative x, y and z</td><td>[3]</td></tr>
<tr><td></td><td>(iii)</td><td>$x \leq 20$, $y \leq 10$ and $z \leq 8$</td><td>M1
A1</td><td>20, 10, 8
Correct inequalities for x, y and z</td><td>[2]</td></tr>
<tr><td colspan="5" align="right">Total =</td><td>16</td></tr>
</table>

<table>
<tr><td colspan="6">TO BE ANSWERED ON INSERT</td></tr>
<tr><td>6</td><td>(i)</td><td>10
½n(n-1)</td><td>B1
B1</td><td>10
1+2+…+(n-1) seen, or equivalent
Check that sum stops at n-1 not n</td><td>[2]</td></tr>
<tr><td></td><td>(ii)(a)</td><td>9
1
2
3
45</td><td>B1

M1

A1</td><td>Their 10 minus 1

1, 2 and 3

45 following from method mark earned cao</td><td>[3]</td></tr>
<tr><td></td><td>(b)</td><td>1+2+3+…+(N-1)
= ½N(N-1), where N = ½n(n-1)
= ¼n(n-1)(½n(n-1) – 1) (given)</td><td>M1

A1</td><td>1+2+3+…+(N-1) or ½N(N-1),
where N = ½n(n-1)
Convincingly achieving the given result</td><td>[2]</td></tr>
<tr><td></td><td>(iii)</td><td>
<table>
<tr><td>M1
Vertices in tree</td><td>M2
Arcs in tree</td><td>M3
Vertices not in tree</td><td>M4
Sorted list</td></tr>
<tr><td></td><td></td><td>ABCDE</td><td>~~D | 2 | E~~
~~A | 3 | E~~
~~A | 4 | C~~
~~C | 5 | D~~
~~B | 6 | E~~
~~B | 7 | C~~
~~A | 8 | B~~
~~C | 9 | E~~</td></tr>
<tr><td>D E</td><td>D | 2 | E</td><td>A B C</td><td></td></tr>
<tr><td>D E A</td><td>D | 2 | E
A | 3 | E</td><td>B C</td><td></td></tr>
<tr><td>D E A C</td><td>D | 2 | E
A | 3 | E
A | 4 | C</td><td>B</td><td></td></tr>
<tr><td>DEACB</td><td>D | 2 | E
A | 3 | E
A | 4 | C
B | 6 | E</td><td></td><td></td></tr>
</table>
</td><td>
M1

M1

M1

A1</td><td>(Order of entries in M1, M2 and M3 does not matter)

Arc A | 3 | E is added to M2, A is added to M1 and deleted from M3

Arc A | 4 | C is added to M2, C is added to M1 and deleted from M3

Arc C | 5 | D is not added to M2 and arc B | 6 | E is added to M2

cao (lists M1, M2 and M3 totally correct, ignore what is done in list M4).</td><td>[4]</td></tr>
<tr><td></td><td>(iv)</td><td>$30\times\left(\frac{500}{100}\right)^4$
= 18750 seconds</td><td>M1

A1</td><td>Or equivalent

cao, with units
(312 min 30 sec or 5 hours 12 min 30 sec)</td><td>[2]</td></tr>
<tr><td colspan="5">Total =</td><td>13</td></tr>
</table>